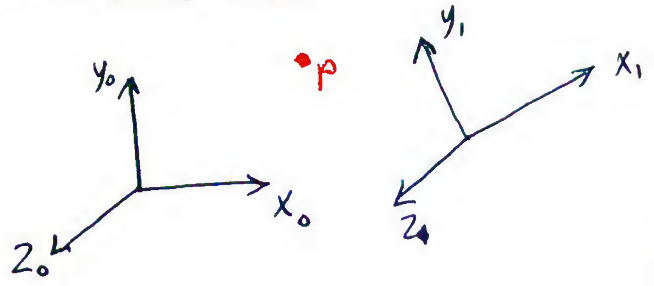
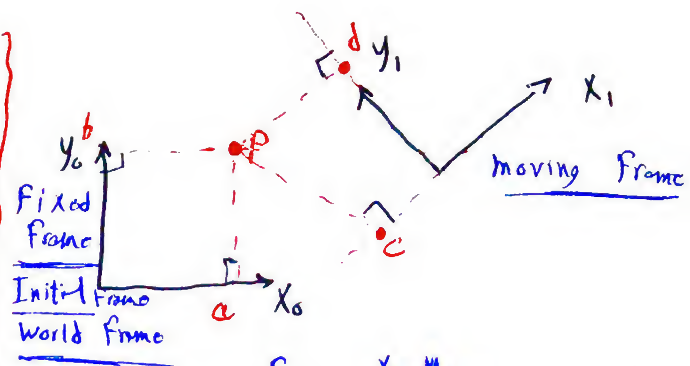
Robotics

Section 3

Rigide motion

- Position Representation
  ↳ reference Frame





$$P = \begin{bmatrix} x \\ y \end{bmatrix}$$

For x_0, y_0

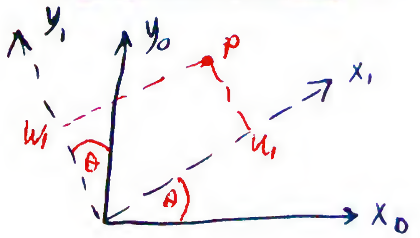
$${}^{0}p = \begin{bmatrix} a \\ b \end{bmatrix} \simeq \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

For x_1, y_1

$${}^{1}p = \begin{bmatrix} c \\ d \end{bmatrix} \simeq \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

- For many applications we can Locate an object to the moving Frame
- we need to transform Location from the moving frame to reference frame

* Rotation representation



$${}^{1}p = u_1 \bar{x}_1 + w_1 \bar{y}_1$$

$$\left.\begin{aligned} \bar{x}_1 &= \cos\theta \, \bar{x}_0 + \sin\theta \, \bar{y}_0 \\ \bar{y}_1 &= -\sin\theta \, \bar{x}_0 + \cos\theta \, \bar{y}_0 \end{aligned}\right\} \rightarrow \text{to matrix}$$

$$\begin{bmatrix} \bar{x}_1 \\ \bar{y}_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \bar{x}_0 \\ \bar{y}_0 \end{bmatrix}$$

$${}^{0}R_1 = R$$

$${}^{0}R_1 = ({}^{1}R_0)^{-1}$$

↳ Transform from 1 to 0

$$\begin{bmatrix} \bar{x}_0 \\ \bar{y}_0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{y}_1 \end{bmatrix}$$

$${}^{1}R_0 = R^{-1}$$

$R \rightarrow$ rotation matrix $\in SO(n)$

$$|R| = \pm 1 \quad \boxed{\text{fev}}$$

$$R^{-1} = R^{T}$$

$${}^{1}p = u_1 \bar{x}_1 + w_1 \bar{y}_1$$

$${}^{1}p = \begin{bmatrix} u_1 & \bar{x}_1 \\ w_1 & \bar{y}_1 \end{bmatrix}$$

$${}^{0}p = {}^{0}R_1 \begin{bmatrix} u_1 & \bar{x}_1 \\ w_1 & \bar{y}_1 \end{bmatrix}$$

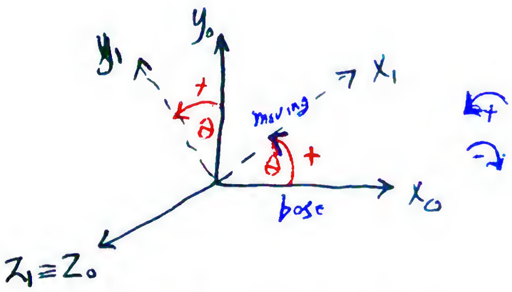
$${}^{0}R_1 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \left[{}^{0}x_1 \;\vdots\; {}^{0}y_1 \right]$$

$${}^{1}R_0 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \left[{}^{1}x_0 \;\vdots\; {}^{1}y_0 \right] = \begin{bmatrix} x_0 \cdot x_1 & x_0 \cdot y_1 \\ y_0 \cdot x_1 & y_0 \cdot y_1 \end{bmatrix}$$

dot product

$${}^{base}R_{moving} = \begin{bmatrix} \;\vdots\; & \;\vdots\; & \\ \end{bmatrix}$$

$$\begin{array}{ccc} {}^{base}x_{new} & {}^{base}y_{new} & {}^{base}z_{new} \end{array}$$



rotation around z_0

$${}^{0}R_1 = R(\theta)$$

$$= \begin{array}{c} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{ccc} {}^{0}x_1 & {}^{0}y_1 & {}^{0}z_1 \end{array} \end{array} = R(z, \theta)$$

$$R(x, \theta) = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \\ \begin{array}{ccc} {}^{0}x_1 & {}^{0}y_1 & {}^{0}z_1 \end{array} \end{array}$$

$$R(y, \theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$